ESTIMATING QUANTILE FUNCTIONS IN TEMPORAL EARTHQUAKE ANALYSIS

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Introduction

In recent decades, substantial research has deepened our understanding of the temporal behavior of seismic activity, consistent with contemporary views on the fractal organization of tectonic structures, fault networks, and hypocenter distributions. These studies emphasize that seismic processes are intrinsically complex and dynamic, often exhibiting “switching” or “shifting” patterns characterized by alternating periods of heightened and diminished earthquake occurrence. Despite these advances, the statistical properties and distributional features of inter-earthquake waiting times remain only partially resolved and, in several aspects, insufficiently explored [1-3]. This gap underscores the need for further investigation aimed at modeling and quantifying the temporal structure of seismicity using more sophisticated statistical tools.

Within modern statistics, the moment problem plays a central role and has extensive applications in mathematics, quantitative finance, economics, and insurance. Although the problem has been studied for more than three centuries, it continues to stimulate significant mathematical interest [4-6]. In this work, we consider the task of approximating and estimating the quantile function under the assumption that the statistical moments are known [7].

We can say that the moment problem has the only solution when the system of equations $\int x^j dF(x) = \int x^j dG(x); j = 0,1,..$, has one solution, F=G.

Material and methods

A range of nonparametric strategies for quantile function estimation has been proposed, including order-statistic methods examined by Harrell and Davis [8] and Bernstein polynomial-based approaches studied by Bolancé et al. [9] and Brewer [10].

The distinct contribution of this work is its applicability to cases where the underlying distribution is poorly known and only statistical moments are available, offering greater flexibility for modeling limited or incomplete data.

Here, we combine the analysis of earthquake waiting times with modern computational methods for quantile-function approximation. Both linear and nonlinear approaches are applied to investigate the temporal distributional patterns. Using waiting-time series from multiple seismic sources, we seek to identify key features of the temporal behavior of seismicity.

To address these challenges, we employ advanced statistical and computational techniques within a custom software framework. Three moment-based models are evaluated for quantile function estimation: one based on frequency moments, another on traditional moments, and a third using transposition moments.

Results

For the analysis of inter-event times (waiting times), we employed data from the Southern California Seismic Catalog (http://www.data.scec.org/ftp/catalogs/.). In particular, records from the Southern California Local Earthquake Catalog were utilized, covering the period from 1932 to 2013. This catalog is considered highly dependable due to its nearly uninterrupted data acquisition throughout the entire observation window.

The dataset was processed using custom software developed at the M. Nodia Institute of Geophysics, Ivane Javakhishvili Tbilisi State University, which enabled the extraction of waiting times between successive earthquake occurrences. Using these waiting-time sequences, we generated an approximate quantile-function estimate based on the first model and compared the obtained results with the well-known Harrell-Davis estimator.

Our current catalog looks like this (Fig. 1):

	A	B	C	D	E	F	G	H	I
1	LONG	LAT	Year	Month	Day	MAG	DEPTH	Hour	Min
2	-115.571	32.552	1975	1	9	2.98	15.7	20	13
3	-115.51	32.673	1975	1	10	2.97	6	20	9
4	-115.234	32.431	1975	1	20	2.78	6	9	25
5	-115.191	32.27	1975	1	27	2.87	6	3	29
6	-115.174	32.454	1975	1	29	2.75	6	2	54
7	-115.476	32.188	1975	2	23	4.26	6	19	7
8	-115.519	32.215	1975	2	27	2.73	6	10	43
9	-115.078	32.219	1975	3	14	2.79	6	14	36
10	-115.037	32.43	1975	3	14	2.78	6	14	58
11	-115.104	32.283	1975	3	14	2.83	6	15	17
12	-115.129	32.318	1975	3	14	2.84	6	15	56
13	-115.103	32.332	1975	3	14	2.86	6	16	5
14	-115.149	32.322	1975	3	14	2.96	6	16	9
15	-115.101	32.343	1975	3	14	2.74	6	16	14
16	-115.232	32.513	1975	3	15	2.86	6	11	46
17	-115.257	32.516	1975	3	16	2.68	6	0	55
18	-115.266	32.455	1975	3	16	2.82	6	3	44
19	-115.086	32.331	1975	3	26	2.75	6	15	56
20	-115.722	32.493	1975	4	5	2.6	3.6	12	29
21	-115.738	32.504	1975	4	6	2.74	5.5	15	48
22	-115.748	32.476	1975	4	6	2.94	1.2	16	59
23	-115.059	32.16	1975	4	28	2.77	6	2	25
24	-114.877	32.28	1975	4	28	2.88	6	2	32
25	-114.845	32.19	1975	4	28	3.07	6	2	49
26	-114.844	32.196	1975	4	28	2.98	6	9	53

Fig. 1. Example of a working catalog.

After that, using a special program created at the M. Nodia Institute of Geophysics of Ivane Javakhishvili Tbilisi State University, we extract the waiting time between earthquakes from this catalog. The resulting file has the following appearance (Fig. 2):

```
3834
952
321
201
232
75
87
724
827
564
287
3308
3612
1790
343
218
```

Fig.2. Waiting time data.

The next step in the research is to estimate the quantile function for the first model based on these data and compare it with the well-known Harrell Davis estimate. For this, we use the following formulas:

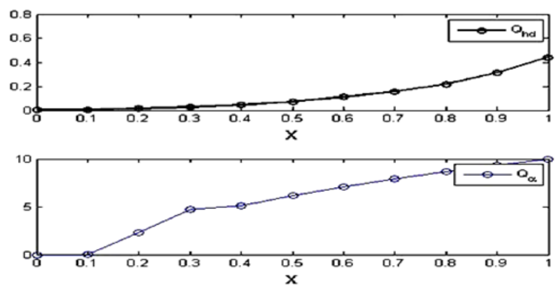
The subsequent analysis is conducted according to the formulations presented below.

$$\widehat{Q_{HD}} = \sum_{i=1}^{n} X_{(i)} \int_{\frac{i-1}{n}}^{\frac{i}{n}} \beta(y, \lfloor \alpha x \rfloor + 1, \alpha - \lfloor \alpha x \rfloor + 1) dy$$

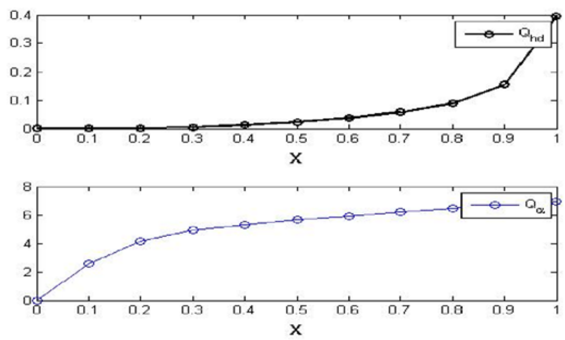
$$Q^{-}_{a,\hat{S}}(x) = \sum_{k=a-[ax]}^{a} \sum_{j=k}^{a} \binom{a}{j} \binom{j}{k} (-1)^{j-k} \sum_{i=1}^{n} \Delta X_{(i)} \left(\frac{n-i+1}{n}\right)^{j}$$

$$\widehat{Q^{-}_{\alpha,}}(x) = \sum_{i=1}^{n+1} \Delta X_{(i)} B_{\alpha}\left(\frac{i-1}{n}, x\right) = \sum_{i=1}^{n} \Delta X_{(i)} [B_{\alpha}\left(\frac{i-1}{n}, x\right) - B_{\alpha}\left(\frac{i}{n}, x\right)]$$

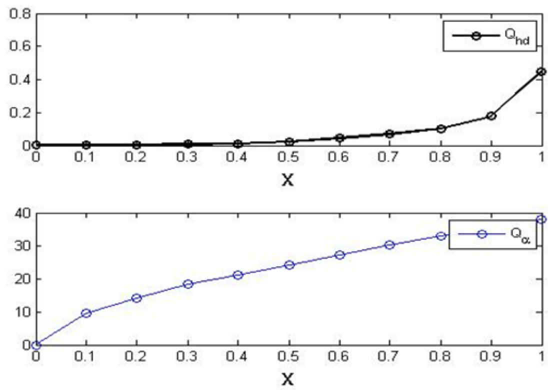
The behavior for different parameters for the first model will be as follows (Fig.3, Fig. 4).



a)



b)



c)

Fig. 3. Comparison of estimates of Harrell Davis and the first model (frequency moments) for the waiting time when a) α =20, n=100 ; b) α =50, n=100 ; c) α =100, n=100 .

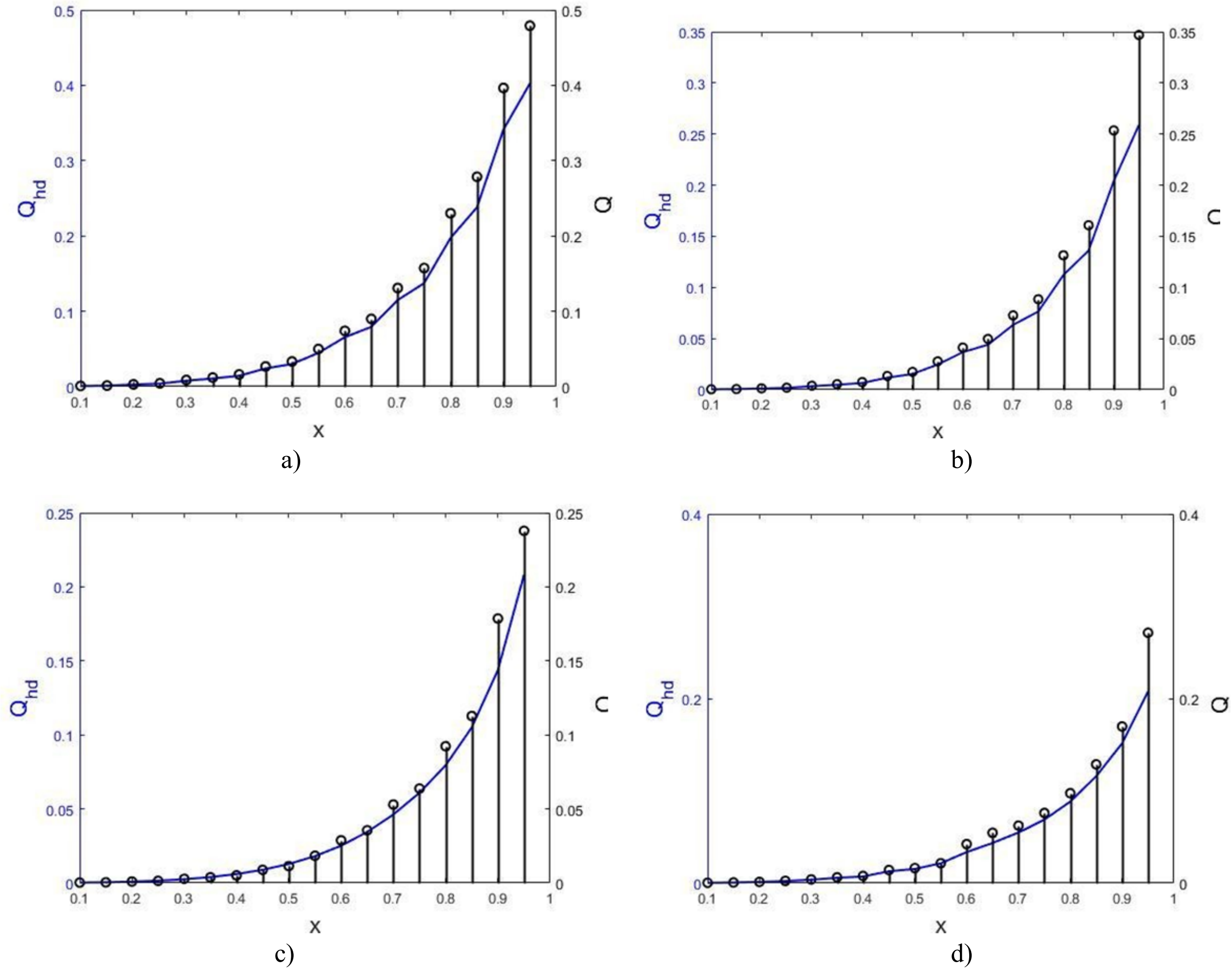


Fig. 4. Harrell Davis estimate for time intervals between earthquakes. Blue curve, black dots estimate of the quantile function, both cases a) α =30, n=100; b) α =30, n=200; c) α =30, n=500; d) α =40, n=200. Southern California Catalog.

Modified versions of the quantile and quantile density function approximations were also considered. The paper presents graphical representations that show that the modified version has a better approximation compared to the conventional model. The robustness theorems were proven, which give us the convergence rate. Modified versions of the approximations were also introduced, whose behavior is better than the original models.

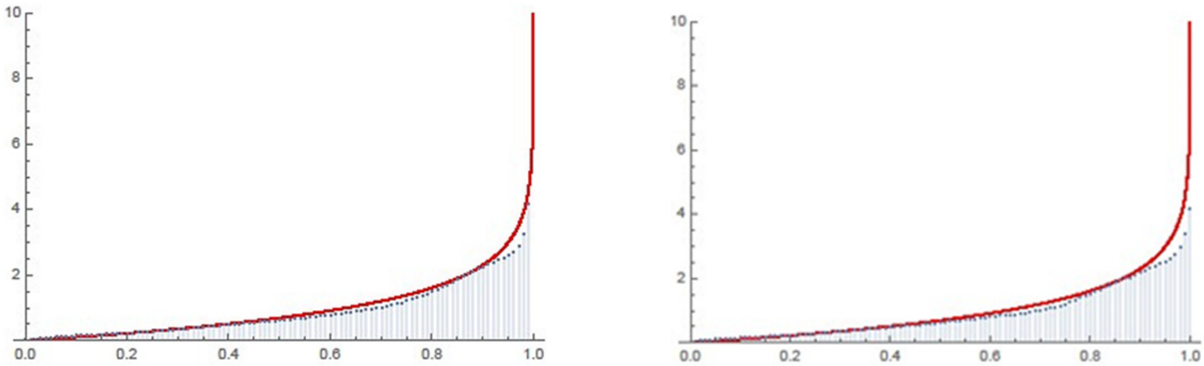
There are several methods for nonparametric estimation of the quantile function. Let us highlight a few, for example, the estimate presented in the paper by Harrell, Davis is a weighted sum of ordinal statistics.

Let us give a graphical representation of the behavior of estimating a quantile and a quantile density function.

Below we can see graphical images of comparison of the quantile function approximation with the Harrell-Davis model (Fig. 5).

The above images show that as n increases, the error decreases. The data allows us to select optimal α values.

At the same time, for comparison, we have done the approximation for various worldwide earthquake catalogues (Kyrgizia, Caucasus and Greece). The results of our analysis is given below (Fig.6, Fig. 7):



a) b)

Fig. 5. Red curve: theoretical quantile function Blue points: approximation of the quantile function by the Harrell-Davis model: a) n=300; b) n=200.

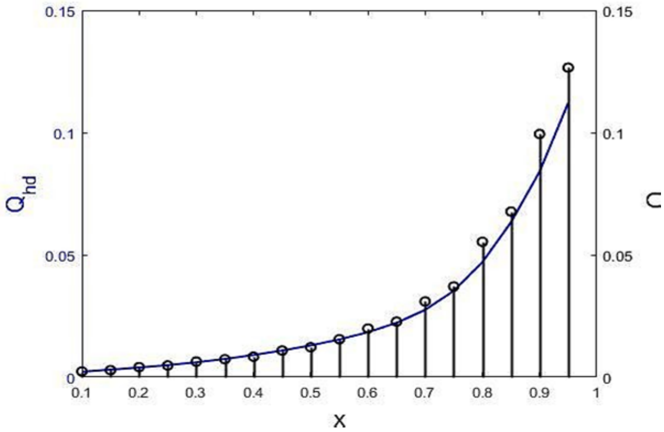
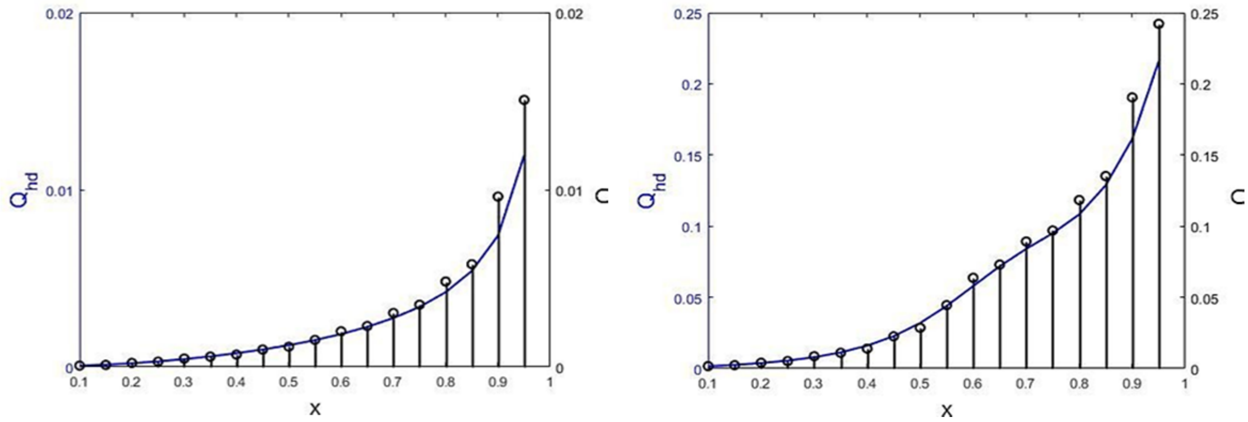


Fig. 6. Approximation of Harrell-Davis model blue curve and our model black points for Kyrgizia earthquake catalogue here $\alpha = 30; n = 300$.



a) b)

Fig.7. Approximation of Harrell-Davis model blue curve and our model black points for a) Caucasus and b) Greece earthquake catalogues in both cases $\alpha = 30; n = 300$.

The results of this study deepen our understanding of seismic behavior and, importantly, provide a foundation for future investigations aimed at clarifying the mechanisms that lead to major earthquakes.

It is worth noting the application of the approach to seismological data. In the paper, based on data from earthquake catalogs of various regions, the time intervals between earthquakes were calculated, and based on the obtained waiting times, arrays were created and the approaches proposed in the paper were applied, and compared with the well-known Harrell-Davis model.

Conclusion

This work analyzes the time intervals between earthquake occurrences using selected datasets to approximate and estimate the quantile function. The proposed methodology has broad applicability, extending to disciplines such as financial mathematics, economics, and insurance. Adopted within a global framework, the study addresses key challenges shared across numerous scientific fields. Its contributions are both theoretical and practical: improving knowledge of the temporal distribution of earthquakes is essential for assessing seismic hazard and for advancing our understanding of the fundamental processes governing earthquake generation.

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ESTIMATING QUANTILE FUNCTIONS IN TEMPORAL EARTHQUAKE ANALYSIS

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Abstract

This work introduces a new framework for estimating and approximating quantile and quantile-density functions based exclusively on known statistical moments. These moments are computed from the waiting times between consecutive earthquakes documented in global seismic catalogs. Three moment-based approximation schemes—frequency moments, classical moments, and transposition moments—are examined to assess their performance. In contrast to traditional methodologies, the proposed approach requires only moment information, providing a streamlined yet reliable strategy for quantile function estimation.

Key Words: Earthquake, Waiting time, Quantile function.

დროითი მიწისძვრის ანალიზით კვანტილის ფუნქციების შეფასება

სბორშჩიკოვი ა., მეფარიძე ე., ჭელიძე თ.

რეზიუმე

ნაშრომში წარმოდგენილია კვანტილის და კვანტილის სიმკვრივის ფუნქციების შეფასების და მიახლოებითი დაანგარიშების ახალი ჩარჩო, რომელიც დაფუძნებულია მხოლოდ ცნობილ სტატისტიკურ მომენტებზე. ეს მომენტები გამოითვლება გლობალურ სეისმურ კატალოგებში დოკუმენტირებული თანმიმდევრული მიწისძვრების მოლოდინის დროების მონაცემებიდან. მათი ეფექტურობის შესაფასებლად განხილულია სამ მომენტზე დაფუძნებული მიახლოების სქემა – სიხშირის მომენტები, კლასიკური მომენტები და ტრანსპოზიციის მომენტები. ტრადიციული მეთოდოლოგიებისგან განსხვავებით, შემოთავაზებული მიდგომა მოითხოვს მხოლოდ მომენტის ინფორმაციას, რაც უზრუნველყოფს კვანტილის ფუნქციის შეფასების გამარტივებულ, მაგრამ საიმედო სტრატეგიას.

საკვანძო სიტყვები: მიწისძვრა, მოლოდინის დრო, კვანტილის ფუნქცია.

ОЦЕНКА ФУНКЦИИ КВАНТИЛЯ С ПОМОЩЬЮ АНАЛИЗА ЗЕМЛЕТРЯСЕНИЙ ВО ВРЕМЕНИ

Сборщиков А., Мепаридзе Е., Челидзе Т.

Реферат

В статье представлена новая методика оценки и аппроксимации функций квантилей и функций плотности распределения квантилей, основанная только на известных статистических моментах. Эти моменты рассчитываются на основе времени ожидания последовательных землетрясений, документированных в глобальных сейсмических каталогах. Для оценки их эффективности рассматриваются три схемы аппроксимации на основе моментов: частотные моменты, классические моменты и транспозиционные моменты. В отличие от традиционных методологий, предлагаемый подход требует только информации о моментах, что обеспечивает упрощенную, но надежную стратегию оценки функции квантиля.

Ключевые слова: Землетрясение, Время ожидания, Функция квантиля.